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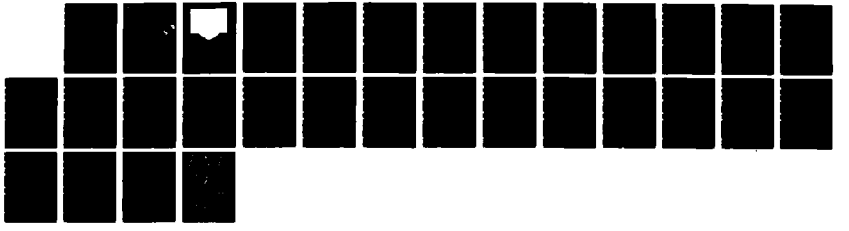

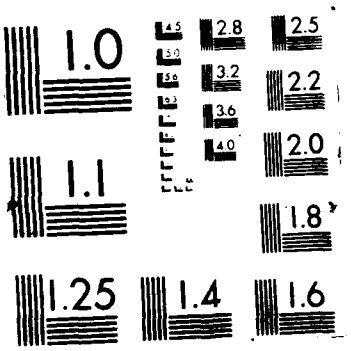
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A simple linear calibration function can be used over a wide concentration range for the Inductively Coupled Plasma (ICP) spectrometer due to its linear responses. The random errors over wide concentration ranges are not constant, and constant variance regression should not be used to estimate the calibration function. Weighted regression techniques are appropriate if the proper weights can be obtained. Use of the calibration curve to estimate the concentration of one or more unknown samples is straightforward, but confidence interval estimation for multiple use of the calibration curve is less obvious. We describe a method for modeling the error along the ICP calibration curve and using the estimated parameters from the fitted model to calculate weights for the calibration curve fit. Multiple and single-use confidence interval estimates are obtained and results along the calibration curve are compared.

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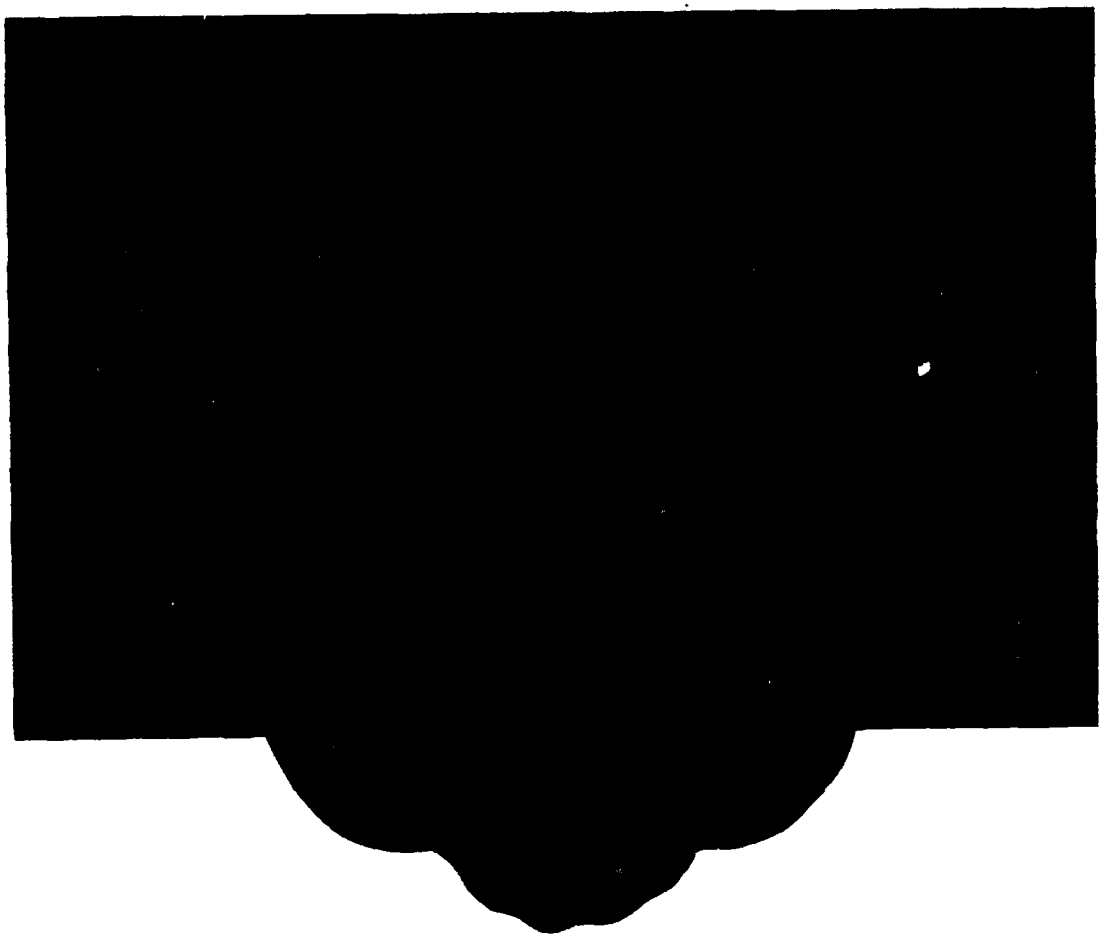

ERROR MODELING AND CONFIDENCE INTERVAL ESTIMATION FOR INDUCTIVELY COUPLED PLASMA CALIBRATION CURVES

By

Robert Watters

Raymond J. Carroll

C. Spiegelman

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DEPARTMENT OF STATISTICS
Chapel Hill, North Carolina

ERROR MODELING AND CONFIDENCE INTERVAL ESTIMATION FOR INDUCTIVELY COUPLED PLASMA CALIBRATION CURVES

Robert L. Watters, Jr.,
National Bureau of Standards

Raymond J. Carroll,
University of North Carolina, Chapel Hill

and

Clifford H. Spiegelman,
National Bureau of Standards

Abstract

A simple linear calibration function can be used over a wide concentration range for the Inductively Coupled Plasma (ICP) spectrometer due to its linear response. The random errors over wide concentration ranges are not constant, and constant variance regression should not be used to estimate the calibration function. Weighted regression techniques are appropriate if the proper weights can be obtained. Use of the calibration curve to estimate the concentration of one or more unknown samples is straightforward, but confidence interval estimation for multiple use of the calibration curve is less obvious. We describe a method for modeling the error along the ICP calibration curve and using the estimated parameters from the fitted model to calculate weights for the calibration curve fit. Multiple and single-use confidence interval estimates are obtained and results along the calibration curve are compared.

Keywords

Calibration, inductively coupled plasma, ICP, error modeling, confidence intervals

Brief

Iteratively weighted error modeling for nonconstant variance ICP calibration curves is examined. Contributions of calibration bands and unknown sample measurement uncertainty intervals are combined to obtain multiple-use confidence intervals. The effects of weighting are examined at both ends of the calibration curve.

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Introduction

A widely used method for determining inorganic elements in aqueous samples is the Inductively Coupled Plasma (ICP) spectrometric technique [1-3]. One of the principal advantages of the ICP technique is its wide range of linear response to analyte concentration. This feature, together with relative freedom from interelement interferences, often allows the analyst to determine a given element in a variety of sample matrices with only infrequent recalibration of the instrument. Although the variability of the calibration process is sometimes assumed to be negligible compared to sample measurement variability, it is desirable to establish procedures whereby this assumption may be conveniently tested. If calibration is indeed a significant contributor to the overall variation of the measurement process, this source of variation should be estimated and included in the overall uncertainty statement.

The calibration curve for ICP spectrometry is an example where weighted regression should be used because the measurements may exhibit non-constant variance (heteroscedasticity). In general, if the calibrated region includes the widest range of concentration for which straight-line response is assured, non-uniform precision of measurement is evident. Agterdenbos [4] approached the problem of "ICP type" calibration curves by dividing the range of calibration into segments that exhibit either constant standard deviation or constant relative standard deviation characteristics. Maessen and Balke [5] also compared the effects of treating ICP calibration curves as combinations of linear and logarithmic segments. The usefulness of simple weighted linear regression was mentioned, but not considered. It was stated by these authors

that no suitable method exists for obtaining the confidence interval for an unknown analyte concentration calculated from a weighted linear regression.

More recently, Bubert and Klockenkamper [6] have considered the effects of heteroscedasticity in ICP and x-ray fluorescence (XRF) calibration curves. They outline a scheme for using established tests for the normality of the distribution of the measured values as well as the homogeneity of variance. An algorithm for evaluating the sources of variance was used, and a weighted linear regression was carried out. Confidence limits for the concentration of an analyzed sample can be calculated using their approach. Schwartz [7] and Oppenheimer, et al. [8] have given methods for weighted regression of calibration curves with nonuniform variance. Confidence intervals for a single determination of an unknown are given using propagation of error in the case of Schwartz [7]. Although the main focus was to treat weighted calibration curve estimation of detection limits, single-use intervals can also be obtained by adapting the approach described by Oppenheimer, et al. [8] to higher concentration levels. A multiple-use procedure has been presented by Garden, et al. [9] that often results in intervals wider than those developed by the methods described in this paper. Garden, et al. acknowledge the conservative nature of their interval estimates, which allow for some nonrandom error that often occurs in spectrometric measurement. We prefer to restrict our estimation of confidence intervals to include random error only. Assessment of systematic errors should be carried out separately, using techniques specifically designed for this task.

We describe here a simple method for characterizing the variance along an ICP calibration curve due to the principal sources of noise in this specific

technique. Our approach is philosophically similar to that of Garden, et al. [9] except that we use calibration data only as a starting point to model the error along the calibration curve. Our method involves standard regression techniques, and can be incorporated into routine analytical schemes due to its simplicity. A flow chart of the procedures is presented in Figure 1. We model both the standard deviation and the variance of instrument noise as a function of sample concentration over the range of straight-line response. We use a quadratic model of concentration as suggested by Oppenheimer, et al. [8] rather than a quadratic model of intensity as used by Bubert and Klockenkamper [6] and Schwartz [7]. We choose to model the error in terms of concentration rather than intensity, since we are dealing with a specific technique whose linear response is well documented. A particular feature of our approach is that each iteration of the noise model fit is itself a weighted fit using estimates from the previous iteration. This procedure differs from the unweighted modeling used by Oppenheimer, et al. [8]. The final set of fitted standard deviations or variances is then used for the weighted regression of the straight-line calibration curve. Confidence limits for concentrations obtained using the calibration curve are calculated in a straightforward manner.

Algorithm

The method we use for characterizing the ICP calibration curve assumes the model

$$Y_{ij} = a + bx_i + error_{ij}$$

where Y_{ij} = measured intensity for

$i = 1, 2, \ldots, I$ calibration standards of concentration x_i,

and $j = 1, 2, \ldots, J \geq 2$ instrumental replicates.

The calibration function is $a + bx$, and the random errors have mean 0, and variance $\sigma^2(x_i)$. The errors are independent and normally distributed. The model for the standard deviation is

$$\sigma(x) = c + dx + ex^2 \tag{2}$$

The estimate of standard deviation s_i at each calibration point, x_i is calculated by using the replicate measurements of each standard solution. These s_i's are then fitted according to model (2) by unweighted least squares to obtain predicted standard deviations, $\hat{\sigma}(x_i)$. We now iterate the fit using weights, $1/\hat{\sigma}^2(x_i)$ to obtain a new set of predicted standard deviations. Each iteration uses weights calculated from the predicted values of the previous step. The procedure stops when successive predicted values agree to within 0.1% relative.

Alternately, we also model variance in a similar manner. The model is

$$\sigma^2(x) = g + hx + kx^2 \tag{3}$$

and the s^2_i's are fitted according to model (3). The iteration is performed as

before except that the variance fit is now weighted by $1/\hat{\sigma}^4(x_i)$. Predicted variances are used to obtain standard deviations at each x_i.

There exists a spectrochemical basis for certain aspects of models (2) and (3). Constant noise sources such as detection electronics account for the concentration-independent term in either model. Shot noise expressed as a variance is proportional to intensity, or in this case the concentration x, which corresponds to the coefficient h in model (3). Source flicker noise corresponds to the second term in model (2) or the third in model (3). A similar partition of noise sources is mentioned in reference [6].

The final set of predicted standard deviations, $\hat{\sigma}_w(x_i)$ obtained from either (2) or (3) is used to calculate weights $1/\hat{\sigma}^2{}_w(x_i)$ for the least squares fit of the calibration function, a + bx. The results of this fit include an estimate for the intercept, $\hat{a}$, the slope, $\hat{b}$, the residual standard deviation $\hat{\sigma}$, and the standard error for the predicted mean at each x, $\hat{\sigma}_{f(x)}$ at concentration x. We assign degrees of freedom, γ for $\hat{\sigma}_{f(x)}$ by

$$\gamma = I - 2.$$

The estimated calibration function is then

$$f(x) = \hat{a} + \hat{b}x.$$

To obtain a concentration value for an unknown sample, we can solve $Y_0 = \hat{a} + \hat{b}x_0$ the mean measured analyte intensity, for x_0; i.e., we invert the calibration function. To obtain an approximate confidence interval for the sample concentration, we combine the confidence interval about the mean sample

intensity with the confidence band about the estimated calibration function. The interval is constructed with probability $1-\alpha$, and the band is constructed with probability $1-\delta$.

Let $t_\gamma(1-\alpha/2)$ be the $1-\alpha/2$ percentile from a t-distribution with γ degrees of freedom, and $F_{2,\gamma}(1-\delta)$ be the $1-\delta$ percentile of an F-distribution with 2 and γ degrees of freedom. The confidence interval for an unknown sample concentration is obtained by measuring its intensity, Y_0, and finding all x's that satisfy the following inequalities:

$$\begin{aligned} f(x) - t_\gamma(1-\alpha/2)\,\hat{\sigma}_w(x)\,\hat{\sigma} \\ - (2F_{2,\gamma}(1-\delta))^{\frac{1}{2}}\,\hat{\sigma}_{f(x)} \;\le\; Y_0 \;\le\; f(x) + t_\gamma(1-\alpha/2)\,\hat{\sigma}_w(x)\,\hat{\sigma} \\ + (2F_{2,\gamma}(1-\delta))^{\frac{1}{2}}\,\hat{\sigma}_{f(x)} \end{aligned} \tag{4}$$

where $t_\gamma(1-\alpha/2)\,\hat{\sigma}_w(x)\,\hat{\sigma}$ is the half-width of the confidence interval about Y, and $(2F_{2,\gamma}(1-\delta))^{\frac{1}{2}}\,\hat{\sigma}_{f(x)}$ is the half-width of the confidence band at x about f(x). Typically, equal values for α and δ are chosen. The derivation of (4) for the homoscedastic case is given in [10], and the resulting confidence band applies to multiple use of the calibration curve. We allow here for the inclusion of weights as determined by the error modeling. The inequalities for calibration followed by a single use [8,11] of the curve reduce to:

$$\begin{aligned} f(x) - t_\gamma(1-\alpha/2)\,((\hat{\sigma}_w(x)\,\hat{\sigma})^2 + (\hat{\sigma}_{f(x)})^2)^{\frac{1}{2}} \\ \le\; Y_0 \;\le \\ f(x) + t_\gamma(1-\alpha/2)\,((\hat{\sigma}_{\cdot\,\cdot})\,\hat{\sigma})^2 + (\hat{\sigma}_{f(x)})^2)^{\frac{1}{2}} \end{aligned} \tag{5}$$

The relationships of the calibration function, confidence interval, and confidence band for the homoscedastic case ($\hat{\sigma}_w(x)$ = constant) are depicted in

Figure 2. The construction of the confidence interval for an unknown sample concentration from replicate measurements of emission intensity is readily discernible from the figure. However, the heteroscedastic case is not so easily represented. We choose to represent the combination of the confidence interval with the confidence band as a widened band whose upper bound, U(x) is the right hand side of the inequalities (4), and the lower bound, L(x) is the left hand side. The resulting band is plotted in Figure 3. The confidence interval for an unknown sample concentration is then simply determined by the intersection of Y with U(x) and L(x). This approach requires that the slope be positive and sufficiently large to ensure that Y crosses the bounds, U(x) and L(x), only once. In most applications of ICP spectrometry, these conditions are fulfilled if analytically useful spectral emission lines are chosen. Curves with negative slope can be treated using simple modifications to (4) and (5). The location of the minimum calibration band width is dependent on the particular set of concentration standards used. The dilution scheme described in the following section results in the appearance of the bands in Figures 2 and 3.

We examine the effects of calibration in various ICP analysis schemes in the following example.

Experimental

The measurements were performed on a sequential ICP spectrometer system with a spectral bandpass of 0.007 nm. The normal ICP spectrometer experimental parameters and operating conditions were used. Various spectral lines were used in this study, but we present the data for Ni at 231.604 nm as being

representative of the characteristics of most ICP spectral lines normally used for analysis. The spectral background at 231.625 nm was measured and subtracted from the peak intensity for each instrumental integration.

A total of ten standard solutions were prepared using volumetric pipets and flasks. However, each aliquot and final dilution volume was weighed so that the calculated concentrations of the calibration standards were based on the gravimetric data. The concentrations of the standards ranged from 0 μg/mL to 5.03 μg/mL, with a reagent blank used as the zero-standard. Vials containing these ten solutions were placed in the autosampler of the spectrometer and the run sequence was programmed. The order of standard solution introduction was randomized, and adequate rinsing for the widest concentration range was accommodated. In this case, the run sequence included a rinse of 45 s in 1% nitric acid in distilled water followed by a washout of 30 s with the next solution to be measured. The first 15 seconds of this washout period take place with a solution uptake rate of 8 mL/min. The sample peristaltic pump returns to the normal rate of 1 mL/min 15 seconds before measurements begin. Spectral intensities are measured by electronic integration of the photomultiplier current for 0.25 seconds.

Results and Discussion

Ten replicate integrations of each solution were recorded, and the average net intensity value, Y_i and standard deviation, s_i for each were calculated. The first analysis of the data included all ten replicates at each of the ten standard concentrations. Since the ICP is often used in laboratories where the sample analysis rate is high, instrumental replicates in such situations may be

limited to as few as four. We therefore examined this case by repeating the fit using only the first four replicates from each set of ten. At this point the data were examined to test the assumption of measurement independence and outliers. The occurrence of outliers and non-random variability caused by the performance characteristics of ICP nebulizers has been previously described [12], and this issue has also been addressed by Garden, et al. [9]. Extensive experience with ICP results for Ni in our laboratory indicated that the set of replicate integrations at 1.01 μg/mL were not representative of normal ICP instrument behavior.

This set of ten replicates has a calculated standard deviation of 15.1, which is the expected level of variability for this concentration level. However, if the ninth integration is excluded, this statistic is 6.2. These data represent a period of measurement wherein the ICP sample delivery and signal detection systems were atypically stable. When only the first four data points are used, the estimated standard deviation (5.5) severely underestimates typical variability at this concentration. Successive integrations over short periods of time often show some degree of drift. However, the interdependence of successive integrations at 1.01 μg/mL, and the existence of the outlier (ninth integration) indicate that the data at this concentration cannot be used to estimate ICP variability. Accordingly, we analyzed the remaining data excluding the entire data set at this concentration. The data are listed in Table 1.

The fit of the estimated standard deviation, $\hat{\sigma}(x)$, for both the 10 and 4 replicate cases according to model (2) yields the parameter estimates listed in Table 2. Results for unweighted, weighted by $1/s^2_i$, and weighted by $1/\hat{\sigma}^2_w(x)$ are given in the table. The largest differences in parameter estimates are

observed between the unweighted case and either of the two weighted fits. Even these differences are relatively small and have little effect on the remainder of the analysis.

The fit of variance, however, is significantly affected by weighting. The fit of estimated variance, $\hat{\sigma}^2(x)$, according to model (3) results in the parameter estimates listed in Table 3 for unweighted, weighted by $1/s^4_i$, and weighted by $1/\hat{\sigma}^4(x)$ fitting. The method of weighting has a significant effect on the parameter estimates. The weights, $1/\hat{\sigma}^2_w(x)$, for the fit of s using model (2) range over one order of magnitude. However, the weights, $1/\hat{\sigma}^4_w(x)$, for the variance fit of model (3) range over two orders of magnitude. Although the high concentration points are important for estimating this fit in the quadratic region, they have extremely low weighting. Therefore, the weighted fitting procedure has difficulty distinguishing between a linear and a quadratic model. Thus, the estimates of the linear and quadratic terms will be determined with large uncertainties, although the fit of either (2) or (3) may be quite adequate.

For limited sets of ICP calibration data, the iterative fitting procedure with $1/\hat{\sigma}^2(x)$ weighting of model (2) or with $1/\hat{\sigma}^4(x)$ weighting of model (3) is preferred, since it appears to be generally applicable and reliable. The iteratively weighted fitting procedure for $\sigma^2(x)$ using model (3) yields a set of predicted standard deviations quite similar to the set obtained by fitting $\sigma(x)$ with model (2). Therefore, we will further examine the fit of model (2) using parameter values from the $1/\hat{\sigma}^2_w(x)$-weighted case in Table 2.

The adequacy of the fit for the 10-replicate case is evident in Table 1, where the observed standard deviations and predicted standard deviations are listed for each of the nine concentrations. The largest difference is at

0.101 μg/mL, where the observed standard deviation value is approximately 70% of the predicted value. This deviation is well within expected instrumental variability at the count level indicated in the table. The predicted standard deviations for the 4-replicate case are also listed in Table 1, and the exhibit reasonable agreement with the 10-replicate estimates. The only significant difference between the two data sets lies in the standard error (se) of the parameter estimates listed in Table 2. It is expected that 10 replicates are more reliable for estimating the fit than 4. This is especially true when problems of interdependence of data arise, as mentioned with regard to the data for the standard at 1.01 μg/mL.

We now examine the fit of the calibration function. A comparison of fitting effects on the calibration curve is presented in Table 4. Weights equal to 1.00, $1/s^2_i$, and $1/\hat{\sigma}^2_w(x)$ are compared. In this case, the $1/s^2$ weights are derived directly from the standard deviation of the calibration data at each concentration (Table 1). The $1/\sigma^2_w$ weights are calculated from the final set of predicted standard deviations using the fit of model (2) for the σ^2_w case. The coefficient and standard error estimates for the 10-replicate data are quite similar regardless of whether $1/s^2_i$ or $1/\hat{\sigma}^2_w(x)$ weights are used. The unweighted case (weights = 1.00) yields slightly different (and erroneous) values for the standard errors. Of course, uncertainty interval estimates for an unknown sample concentration should not be calculated for the unweighted case since homoscedasticity would have to be assumed. Data in table 1 indicate that no single value for the standard deviation is suitable over the range of calibration.

Using only 4 replicates causes a small change in the slope and a relatively larger change in the intercept, especially for the $1/s^2_i$-weighting

case. However, the standard errors for these estimates are large enough to minimize the significance of these differences. In practical applications where the chief concern is trace analysis near the intercept, a more limited range of calibration is appropriate and the standard error of the intercept is likely to be smaller. In terms of the standard error of the estimated coefficients, the 4-replicate data represents a minor deterioration in the variability of the fit, except for the unweighted case.

Although the spectral background near a number of ICP spectral lines for a variety of elements may exhibit more complex structure, the background near the 231.604-nm line of Ni is quite flat. Therefore, net intensity measurements for Ni could be fitted to a zero-intercept calibration model, as is indicated by the intercept estimates and their standard errors listed in Table 4.

Confidence interval estimation was carried out using $\alpha = 0.10$. The components of inequalities (4) were evaluated for the 10-replicate case and are listed in Table 1. It is evident that the uncertainty interval for the unknown sample measurement and the calibration uncertainty band contribute almost equally to the total uncertainty. Ignoring either source of variability will result in a significant underestimation of the random error in a sample analysis.

Comparisons of final confidence intervals obtained for the concentration of an unknown sample are presented in Figures 4 and 5. The average value for the last four integrations at 0.101 μg/mL and at 5.03 μg/mL was used as the mean intensity for each of two unknown sample measurements. Multiple-use confidence intervals were calculated for 10 and 4-replicate calibration, and for weighting with $1/\hat{\sigma}^2_w(x)$ and 1.00 as weights at each x_i. Figure 4 depicts data at the low end of the calibration curve where the true concentration of

the unknown is 0.1006 μg/mL. At this end of the calibration curve, each multiple-use interval is 41% wider than its corresponding single-use interval. When comparing intervals for 10 and 4-replicate cases, it is useful to consider which elements in the inequalities (4) differ. Different parameter estimates for model (2) lead to different sets of predicted standard deviations. These are used for the weighted calibration curve fit, so that the f(x) term in (4) is affected, as is the standard deviation of the fit, $\hat{\sigma}$. The predicted standard deviation at the unknown concentration, $\hat{\sigma}_w(x_i)$, is also obtained from the fit of model (2), so that this term will also be affected. For the data plotted in Figure 4, the predicted value for $\hat{\sigma}_w(x_i)$ for 4 replicates was less than that for 10-replicate calibration. Again, this phenomenon is due to short-term stability of 4 replicate integrations that leads to lower estimates of variability. This underscores the need for caution when sample throughput demands force a reduction in the number of instrumental replicates taken.

An important difference in interval width exist between the weighted and unweighted cases. In the latter case, the value of $\hat{\sigma}_w(x_i)$ is 1.0 and at the low end of the calibration curve, $\hat{\sigma}$ is appreciably larger for the unweighted fit than the weighted fit. The product, $\hat{\sigma}_w(x_i)\ \hat{\sigma}$, is therefore slightly larger for the unweighted case. This causes a small over-estimation of confidence interval widths at the low end of the calibration curve.

Intervals for the high end of the calibration curve are depicted in Figure 5, where the true value of the unknown concentration is 5.03 μg/mL. In this case, the predicted value for $\hat{\sigma}_w(x_i)$ for 4 replicates is greater than that for 10, causing the 4-replicate intervals to be wider than the corresponding 10-replicate intervals. At this end of the calibration curve, however, the interval widths for fitting with weights 1.0 significantly under-estimate the

true interval widths as estimated by the weighted cases. The larger variance at the high end of the calibration curve, as estimated by $\hat{\sigma}_w(x_i)$ accounts for this difference. At this end of the calibration curve, each multiple-use interval is approximately 63% wider than its corresponding single-use interval.

In summary, estimation of the error along the calibration curve is important for weighted regression. Error modeling is more stable than using the standard deviations of the calibration standard measurements themselves. The iterative weighted fitting procedure is applicable to standard deviation modeling, and is the preferred approach in variance modeling. Clearly, if heteroscedasticity is ignored, confidence intervals will be too narrow at the high end and too wide at the low end of the ICP calibration curve. The magnitude of these effects will depend on the particular dilution scheme used to make the calibration standard solutions. Estimates of both single and multiple-use confidence intervals differ significantly. Therefore, care should be taken in applying these procedures to a particular analysis scheme.

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Table 1. ICP data for Ni at 231.604 nm using 10 replicates, $1/\hat{\sigma}^2_w(x)$ weighting of model (2), and $1/\hat{\sigma}^2_w(x)$ weighting of the calibration function. Values at each concentration are listed for observed standard deviation, s_i(obs); predicted standard deviation for 10 replicates, s(10 reps) and 4 replicates, s(4 reps); sample measurement interval, SI; calibration uncertainty band width, CB; and total uncertainty band, Total.

Concentration (μg/mL)	Intensity (counts)	s_i (obs)	s (10 reps)	s (4 reps)	SI	CB	Total
0.00	11.33	8.54	7.88	6.97	17.95	10.54	[illegible]
0.0101	16.60	7.88	7.98	7.03	18.17	10.49	[illegible]
0.0251	37.92	9.06	8.12	7.12	18.50	10.43	[illegible]
0.0503	57.00	8.46	8.36	7.28	19.05	10.33	[illegible]
0.101	149.88	6.13	8.84	7.58	20.14	10.19	[illegible]
0.251	369.24	11.57	10.25	8.50	23.34	10.13	[illegible]
0.503	763.36	11.94	12.48	10.02	28.42	11.20	[illegible]
2.51	3688.46	26.24	25.42	22.25	57.89	37.85	[illegible]
5.03	7431.08	29.12	29.30	37.55	66.74	76.67	[illegible]

Table 2. Coefficient and standard error (SE) estimates for fit of model (2) using 9 standard concentrations.

	10 Replicates		4 Replicates	
Coefficient	Estimate	SE	Estimate	SE
Unweighted:				
$\hat{c}$	7.78	0.56	6.80	1.29
$\hat{d}$	10.28	1.17	7.25	2.67
$\hat{e}$	-1.20	0.24	-0.26	0.55
Weighted by $1/s^2_i$:				
$\hat{c}$	7.88	0.56	7.00	1.42
$\hat{d}$	9.66	2.63	5.84	6.82
$\hat{e}$	-1.07	0.58	0.046	1.62
Weighted by $1/\hat{\sigma}^2_w(x)$:				
$\hat{c}$	7.88	0.56	6.97	1.41
$\hat{d}$	9.69	2.59	6.07	6.56
$\hat{e}$	-1.08	0.57	0.0025	1.60

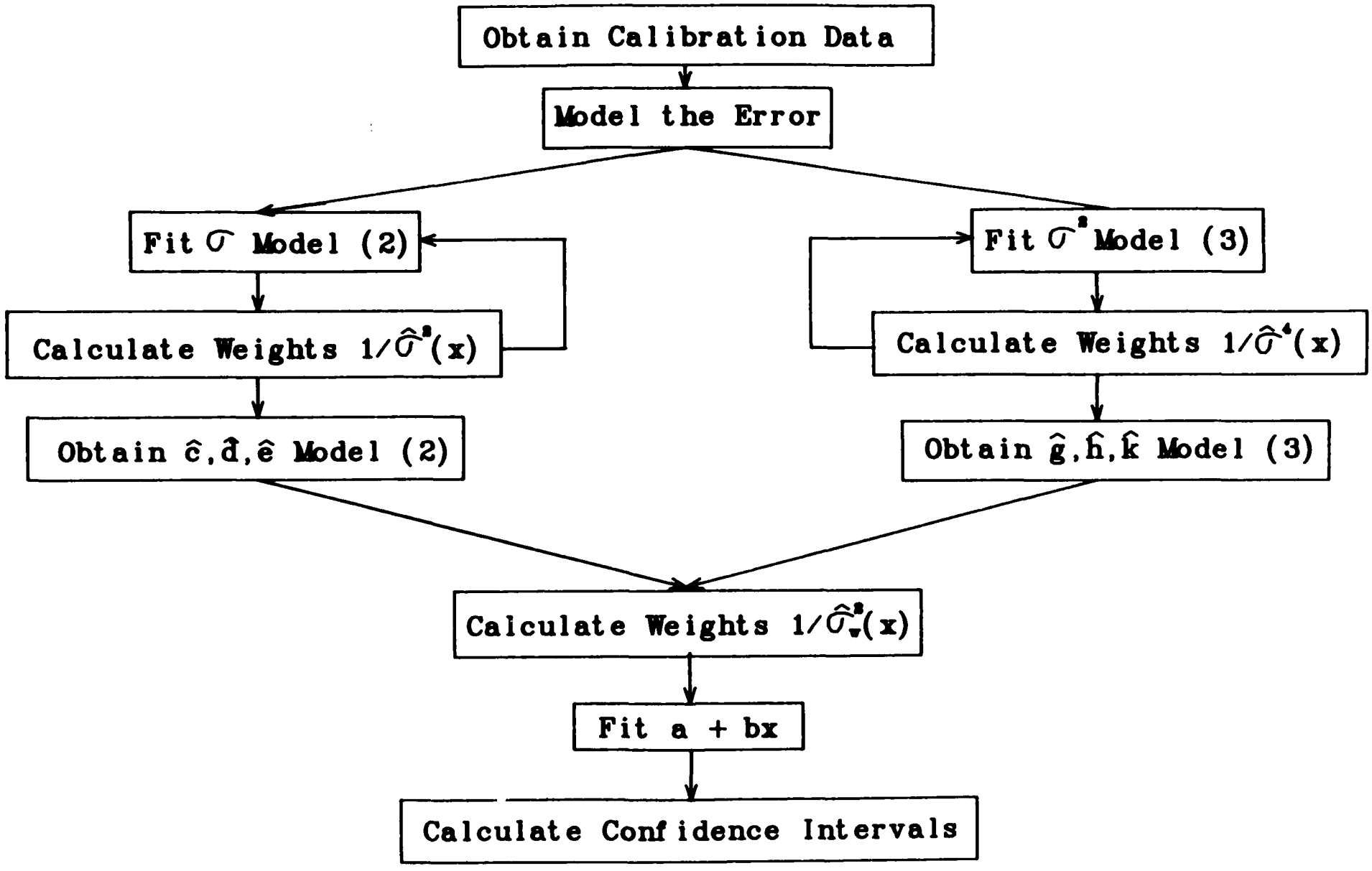
Table 3. Coefficient and standard error (SE) estimates for fit of model (3) using 9 standard concentrations.

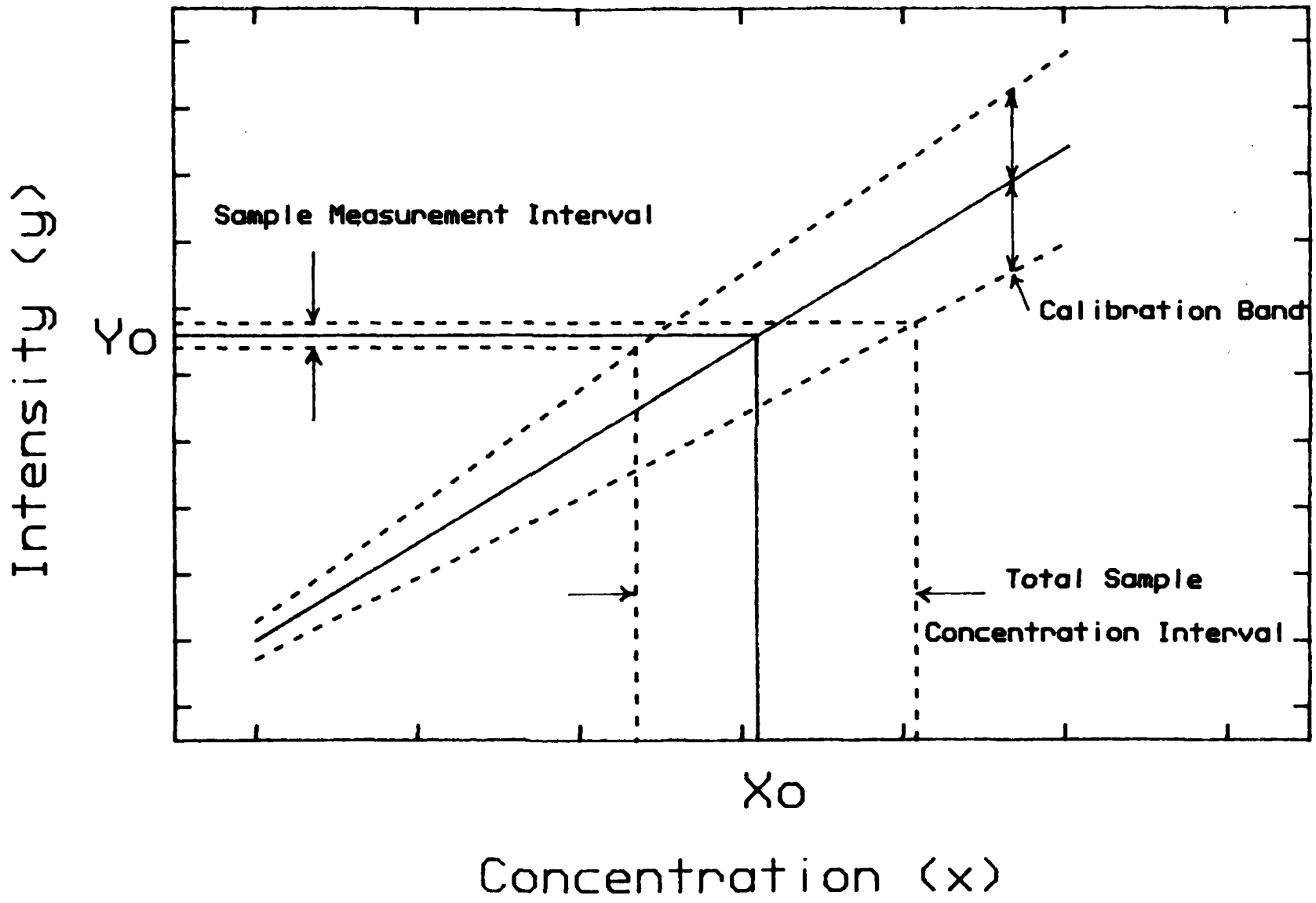
	10 Replicates		4 Replicates	
Coefficient	Estimate	SE	Estimate	SE
Unweighted:				
$\hat{g}$	45.6	15.1	51.0	20.3
$\hat{h}$	332.6	31.2	135.5	41.8
$\hat{k}$	-34.2	6.42	23.9	8.6
Weighted by $1/s^4_i$:				
$\hat{g}$	71.6	22.8	65.1	23.0
$\hat{h}$	-46.6	254.6	5.1	189.8
$\hat{k}$	62.8	62.2	57.2	62.9
Weighted by $1/\hat{\sigma}^4_w(x)$:				
$\hat{g}$	60.6	22.9	60.6	22.9
$\hat{h}$	42.7	148.7	42.7	148.7
$\hat{k}$	50.4	56.7	50.4	56.7

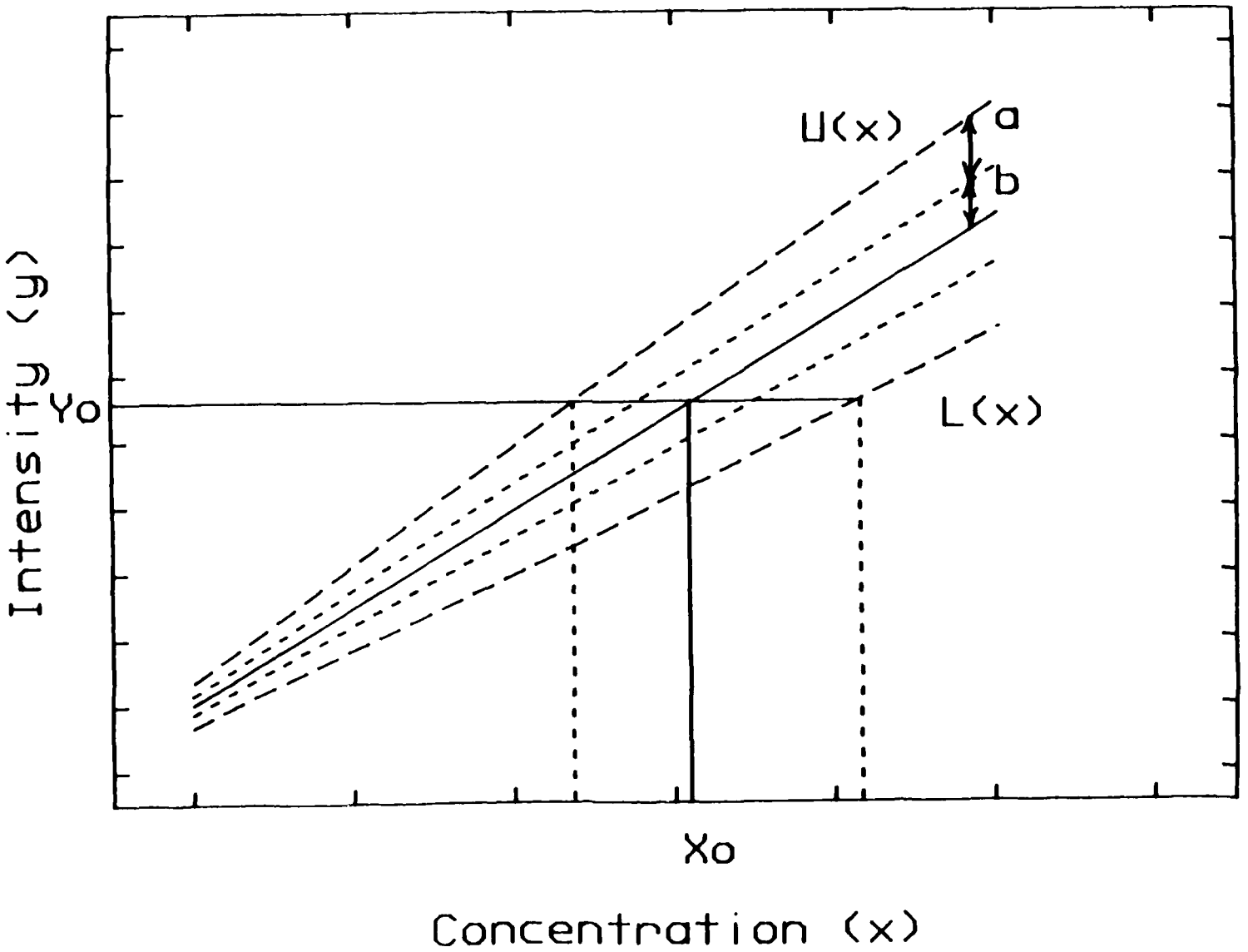
Table 4. Effects of weighting method on the calibration curve fit, a + bx.

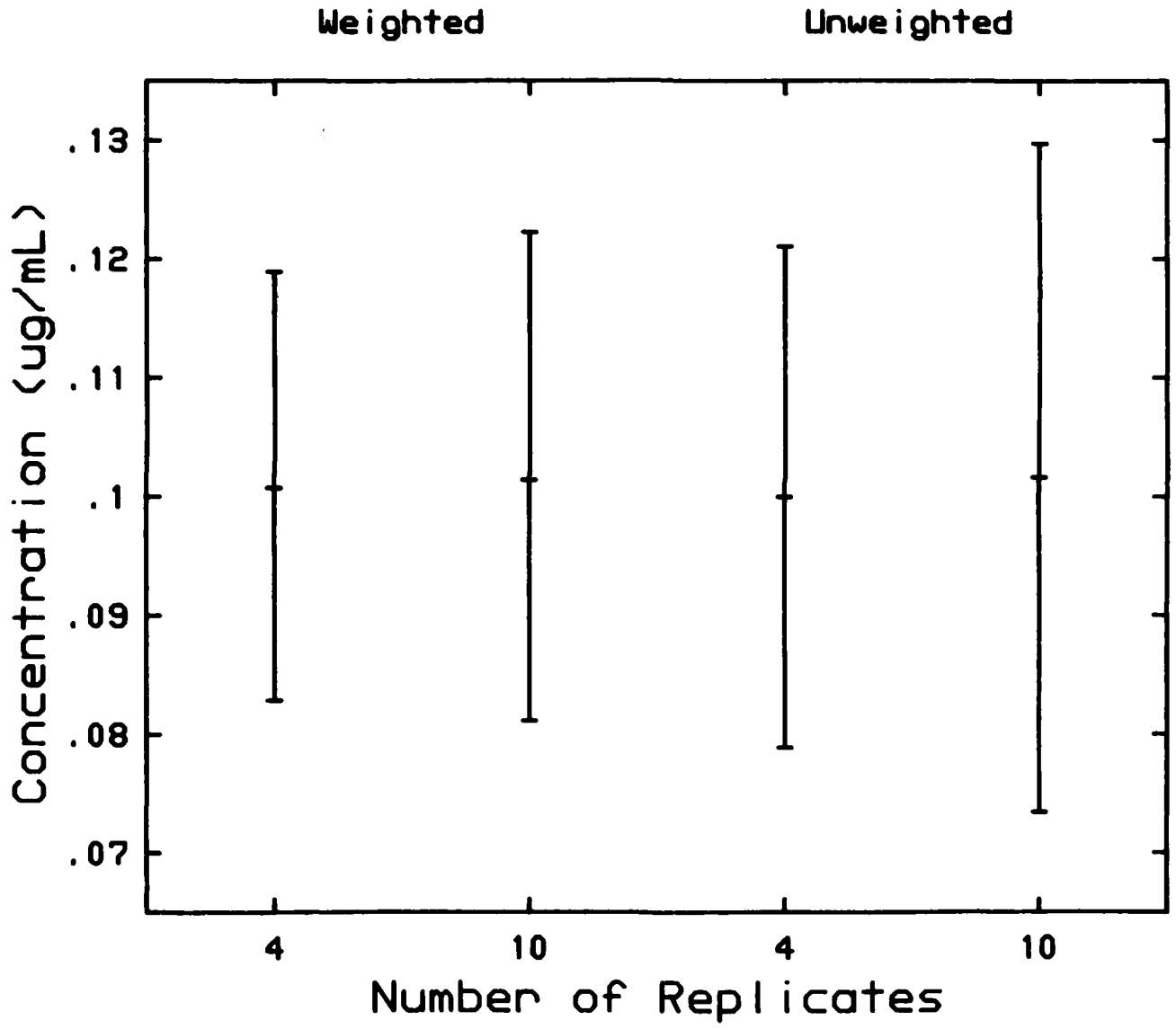
		10 Replicates		4 Replicates	
Case	Coefficient	Estimate	SE	Estimate	SE
Unweighted	a	0.62	5.61	2.87	4.21
	b	1476.04	2.98	1477.69	2.24
Weighted by	a	0.95	3.95	-0.89	3.71
$1/s^2_i$	b	1476.66	6.11	1481.12	9.01
Weighted by	a	0.94	4.13	1.52	3.69
$1/\hat{\sigma}_w^2(x)$	b	1476.30	6.16	1480.38	7.00

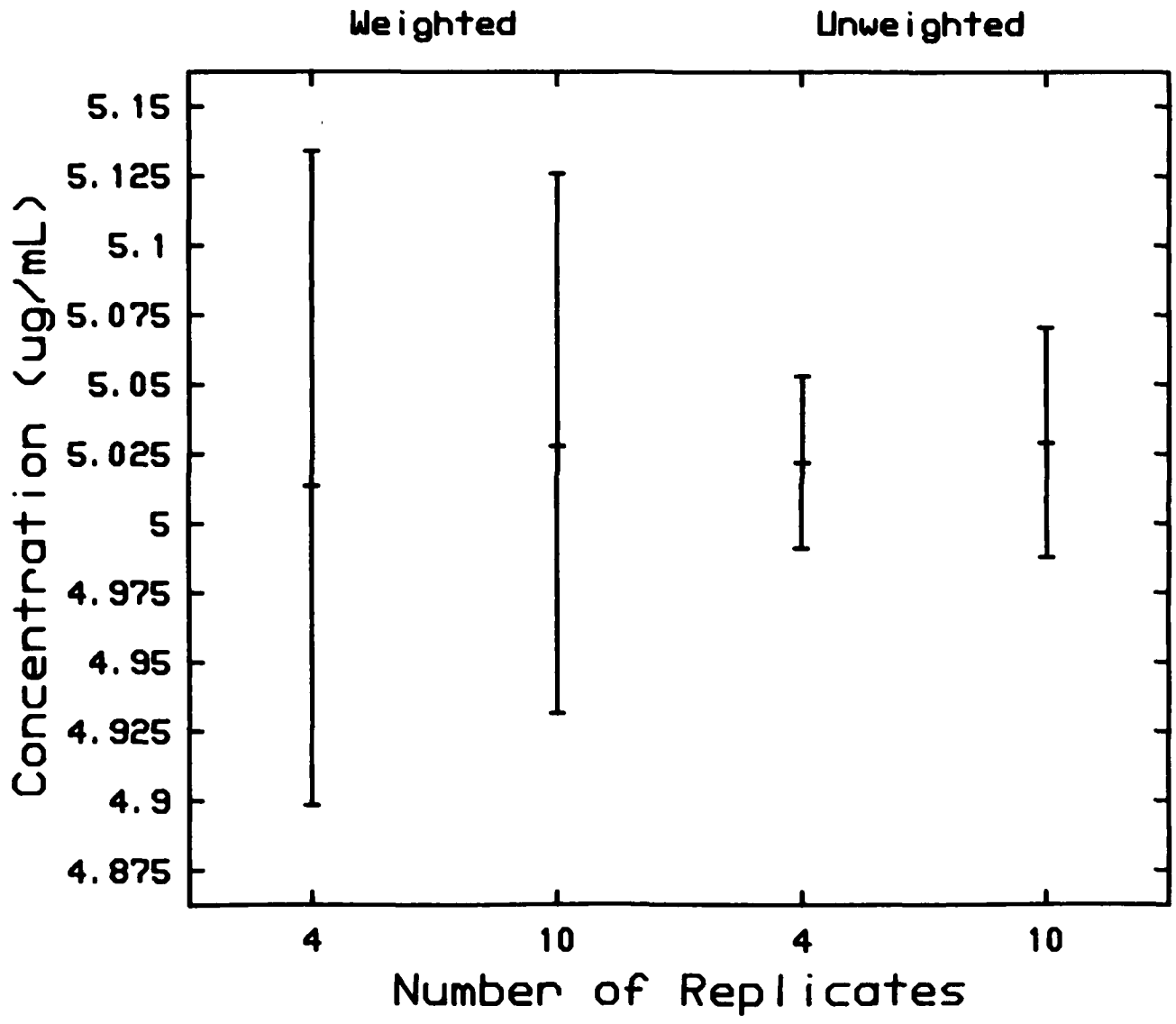
List of Figures


Obtain Calibration Data
Model the Error
Fit σ Model (2)
Calculate Weights $1/\hat{\sigma}^2(x)$
Obtain ĉ, d̂, ê Model (2)
Fit σ^2 Model (3)
Calculate Weights $1/\hat{\sigma}^4(x)$
Obtain ĝ, ĥ, k̂ Model (3)
Calculate Weights $1/\hat{\sigma}_w^2(x)$
Fit a + bx
Calculate Confidence Intervals


Intensity (y)
Yo
Sample Measurement Interval
Calibration Band
Total Sample
Concentration Interval
Xo
Concentration (x)


U(x)
a
b
L(x)
Yo
Intensity (y)
Xo
Concentration (x)


Weighted
Unweighted
.13
.12
.11
.1
.09
.08
.07
Concentration (ug/mL)
4
10
4
10
Number of Replicates


Weighted
Unweighted
5.15
5.125
5.1
5.075
5.05
5.025
5
4.975
4.95
4.925
4.9
4.875
Concentration (ug/mL)
4
10
4
10
Number of Replicates

END

DATE

FILM

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